Activity description

Students use differentiation to find maximum and minimum points in polynomial functions in order to solve a variety of real-life problems.

Suitability

Level 3 (Advanced)

Time

1–2 hours

Resources

Student information sheet
Optional: slideshow

Equipment

Optional: graphic calculators

Key mathematical language

Function, maximum, minimum, differentiation, model.

Notes on the activity

Students need to be able to:

- differentiate polynomials
- solve linear and quadratic equations
- sketch curves.

It is recommended that you use the Nuffield activity ‘Stationary points’ before this activity.

The Nuffield activity ‘Maximising and minimising’ can be used later, possibly as an assignment, to check students’ ability to use the methods to devise and solve a problem of their own.

During the activity

The slideshow starts with a reminder of the derivatives of basic functions, and of the main points from the ‘Stationary points’ activity. It includes the same examples as the students’ information sheet. These can be used to demonstrate the methods and encourage class discussion.

The worksheet will give students practice in using calculus to solve a wide variety of problems set in real contexts.

Points for discussion

Check whether students understand the reasoning behind some of the methods used. For example, ask them questions such as:

'Why is $\frac{dy}{dx} = 0$ at a turning point?'

'Why is $\frac{d^2y}{dx^2}$ negative at a maximum point?'

Explain that the function used in a problem can arise from modelling, as in the velocity and profit problems, or be set up from first principles as in the case of the enclosure and hot water tank problems.

Extensions

Questions 12–14 require students to create the functions from first principles, and are therefore more challenging. These could be omitted by less able students, and kept as extensions for the more able students.

Answers

1 8 m s^{-1}

2 20 m

3 40 m

4 Maximum 53 p after 1 day, minimum 45 p after 3 days

5a 25.5 m **b** 650.25 m^2

6a 25.5 m **b** 1300.5 m^2

7a 25.5 m **b** 2601 m^2

8b 10 cm **c** 18 000 cm^3

9a x = 20 giving 144 000 cm^3

b x = 7.85 giving 8 450 cm^3 (3 sf)

10b 7.37 cm **c** 326 cm^2 (3 sf)

11 x = 9.28 giving 259 cm^2 (3 sf)

12 r = 4.30 giving 349 cm^2 (3 sf)

13 r = 5.42 giving 554 cm^2 (3 sf)

14 r = 0.564 m, giving width 1.13 m, height 0 m
That is, minimum perimeter is when the shape is a circle of radius 0.564 m with perimeter 3.54 m.